

Infrared and extended on-mass-shell renormalization of two-loop diagrams

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Abstract

Using a toy model Lagrangian we demonstrate the application of both infrared and extended on-mass-shell renormalization schemes to multiloop diagrams by considering as an example a two-loop self-energy diagram. We show that in both cases the renormalized diagrams satisfy a straightforward power counting.

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I. INTRODUCTION

The generalization of mesonic chiral perturbation theory [1, 2, 3] to the one-nucleon sector originally turned out to be problematic due to the fact that higher-order loops contribute to lower-order calculations [4] (for a recent review see, e.g., Ref. [5]). This problem was first overcome in the framework of the so-called heavy-baryon approach to chiral perturbation theory [6, 7]. More recently, it has been realized that the power counting can also be restored in the original manifestly Lorentz-invariant formulation [8, 9, 10, 11, 12, 13, 14, 15, 16]. In this context, the infrared (IR) regularization of Becher and Leutwyler [10] (based on ideas of Ref. [8, 9]) is the most widely used renormalization scheme¹ for one-loop diagrams [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. In Ref. [33] it has been suggested that the IR renormalization can also be generalized to multiloop diagrams. In a recent paper [34], we have reformulated the IR renormalization in a form analogous to the extended on-mass-shell (EOMS) scheme of Ref. [14]. In its new formulation the IR renormalization can also be directly applied to diagrams involving resonances as well as multiloop diagrams. In Ref. [35] the application of the EOMS scheme to two-loop diagrams has been illustrated for the nucleon self-energy “rainbow” diagram in the chiral limit. The aim of this work is to demonstrate the application of the reformulated IR renormalization [34] to two-loop diagrams. For comparison the results of the EOMS renormalization are also given.

II. LAGRANGIAN AND POWER COUNTING

In order to simplify the calculations and—without loss of generality—to make the renormalization procedure more transparent, we suppress the complicated spin and chiral structure of baryon chiral perturbation theory and restrict ourselves to the toy model Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \pi \partial^\mu \pi - M^2 \pi^2) + \frac{1}{2}(\partial_\mu \Psi \partial^\mu \Psi - m^2 \Psi^2) - \frac{g}{4} \pi^2 \Psi^2 + \mathcal{L}_1, \quad (1)$$

where π and Ψ are scalar particles with masses M and m , respectively, $M \ll m$, and $g > 0$. The Lagrangian \mathcal{L}_1 contains all interaction terms which are consistent with Lorentz invariance and the invariance under the transformations $\pi \rightarrow -\pi$ and $\Psi \rightarrow -\Psi$.² Both the IR and the EOMS renormalization are constructed such that, after subtraction, Feynman diagrams have certain “chiral” orders D , which are determined by the following power counting: if Q stands for small quantities such as the π mass M , small external four momenta of π or small external three-momenta of Ψ , then the $\pi\Psi$ interaction explicitly shown in Eq. (1) counts as Q^0 , the Ψ propagator as Q^{-1} , the π propagator as Q^{-2} , and a loop integration in n dimensions as Q^n , respectively.

¹ Since the IR regularization is in fact a renormalization scheme we will further on refer to it as *IR renormalization*.

² No particular physical content is meant by imposing these discrete symmetries.

III. APPLICATION TO THE TWO-LOOP SELF-ENERGY

As a specific example of a two-loop diagram, we consider the Ψ self-energy diagram of Fig. 1 (a),

$$-i\Sigma_\Psi(p) = -\frac{i}{2}g^2 J_{\pi\pi\Psi}(0,0,p), \quad (2)$$

where $1/2$ is a symmetry factor and

$$J_{\pi\pi\Psi}(0,0,p) \equiv \left(\frac{i}{(2\pi)^n}\right)^2 \iint \frac{d^n k_1 d^n k_2}{(k_1^2 - M^2 + i0^+)(k_2^2 - M^2 + i0^+)[(p + k_1 + k_2)^2 - m^2 + i0^+]} \quad (3)$$

with n denoting the number of space-time dimensions. Using the above power counting, we assign the order Q^{2n-5} to the diagram of Fig. 1 (a). A dimensional counting analysis [36] (see the Appendix for an illustration) suggests that, if first $M \rightarrow 0$ and then $p^2 - m^2 \rightarrow 0$, the self-energy Σ_Ψ can be written as

$$\Sigma_\Psi = F(p^2, m^2, M^2, n) + M^{n-2}G(p^2, m^2, M^2, n) + M^{2n-4}H(p^2, m^2, M^2, n). \quad (4)$$

The functions $F(p^2, m^2, M^2, n)$, $G(p^2, m^2, M^2, n)$, and $H(p^2, m^2, M^2, n)$ can be expanded in nonnegative integer powers of M^2 and, in the following, will be analyzed in detail.

First, we consider $F(p^2, m^2, M^2, n)$. The Taylor expansion of F in M^2 is obtained by expanding the integrand of Eq. (3) in M^2 and interchanging summation and integration:

$$F(p^2, m^2, M^2, n) = -\frac{g^2}{2} \sum_{i,j=0}^{\infty} \frac{(M^2)^{i+j}}{(2\pi)^{2n}} \iint \frac{d^n k_1 d^n k_2}{(k_1^2 + i0^+)^{1+i}(k_2^2 + i0^+)^{1+j}[(p + k_1 + k_2)^2 - m^2 + i0^+]}. \quad (5)$$

The integrals in Eq. (5) can be written as [36]

$$\begin{aligned} & \iint \frac{d^n k_1 d^n k_2}{(k_1^2 + i0^+)^{1+i}(k_2^2 + i0^+)^{1+j}[(p + k_1 + k_2)^2 - m^2 + i0^+]} = \\ & \sum_{l=0}^{\infty} (p^2 - m^2)^l f_{ij,l}^{(1)}(m^2, n) + (p^2 - m^2)^{2n-5-2i-2j} \sum_{l=0}^{\infty} (p^2 - m^2)^l f_{ij,l}^{(2)}(m^2, n). \end{aligned} \quad (6)$$

Inserting Eq. (6) into Eq. (5) and taking into account that $p^2 - m^2$ counts as order Q , we see that the part which is proportional to noninteger powers of $p^2 - m^2$ (for noninteger n) is of order Q^{2n-5} and therefore satisfies the power counting. The other part is a Taylor expansion in M^2 and $(p^2 - m^2)$ and contains terms which violate the power counting. This Taylor series can be obtained by formally expanding the integrand of the original integral of Eq. (3) in M^2 and $p^2 - m^2$ [14] and interchanging summation and integration. Those terms in the Taylor series which do not violate the power counting contain IR divergences. In the IR renormalization, all terms of the so obtained Taylor series must be subtracted from the original integral. [These subtractions are generated by the counterterm diagrams of Fig. 1 (c)]. However, before the subtraction all IR divergences need to be removed from the subtraction terms [34]. This is necessary to ensure that the subtracted expression does not contain IR divergences. In the following, the omission of IR divergences from the subtraction terms in the IR renormalization is always implied. In the EOMS scheme we only need to subtract those terms which violate the power counting (these terms are free of infrared divergences)—in the present case all terms

of order Q^2 or less [14]. Up to and including order Q^2 the subtraction terms of the EOMS renormalization scheme are given by

$$\Delta F^{\text{EOMS}} = \Delta_{0,0} + \Delta_{1,0}(p^2 - m^2) + \Delta_{2,0}(p^2 - m^2)^2 + \Delta_{0,1}M^2, \quad (7)$$

where the coefficients are given by

$$\begin{aligned} \Delta_{0,0} &= -\frac{g^2(m^2)^{n-3}}{2(4\pi)^n} \frac{\Gamma(3-n)\Gamma(n/2-1)\Gamma(2-n/2)\Gamma(n/2-1)\Gamma(2n-5)}{\Gamma(3n/2-3)\Gamma(n-2)}, \\ \Delta_{1,0} &= -\frac{g^2(m^2)^{n-4}}{2(4\pi)^n} \frac{\Gamma(4-n)\Gamma(n/2-1)\Gamma(3-n/2)\Gamma(n/2-1)\Gamma(2n-6)}{\Gamma(3n/2-6)\Gamma(n-2)}, \\ \Delta_{2,0} &= -\frac{g^2(m^2)^{n-5}}{4(4\pi)^n} \frac{\Gamma(5-n)\Gamma(n/2-1)\Gamma(4-n/2)\Gamma(n/2-1)\Gamma(2n-7)}{\Gamma(3n/2-3)\Gamma(n-2)}, \\ \Delta_{2,0} &= \frac{g^2(m^2)^{n-4}}{2(4\pi)^n} \frac{\Gamma(4-n)\Gamma(n/2-2)\Gamma(3-n/2)\Gamma(n/2-2)\Gamma(2n-7)}{\Gamma(3n/2-4)\Gamma(n-4)}. \end{aligned}$$

The corresponding subtraction terms of the IR renormalization scheme read³

$$\Delta F^{\text{IR}} = \Delta F^{\text{EOMS}} + \delta_{3,0}(p^2 - m^2)^3 + \delta_{1,1}(p^2 - m^2)M^2 \quad (8)$$

with

$$\begin{aligned} \delta_{3,0} &= -\frac{g^2(m^2)^{n-6}}{12(4\pi)^n} \frac{\Gamma(6-n)\Gamma(n/2-1)\Gamma(5-n/2)\Gamma(n/2-1)\Gamma(2n-8)}{\Gamma(3n/2-3)\Gamma(n-2)}, \\ \delta_{1,1} &= \frac{g^2(m^2)^{n-5}}{2(4\pi)^n} \frac{\Gamma(5-n)\Gamma(n/2-2)\Gamma(4-n/2)\Gamma(n/2-2)\Gamma(2n-8)}{\Gamma(3n/2-4)\Gamma(n-4)}. \end{aligned}$$

Next, let us investigate $G(p^2, m^2, M^2, n)$ which is identified from Eq. (3) by rescaling $k_1 \mapsto Mk_1, k_2 \mapsto k_2$ and $k_1 \mapsto k_1, k_2 \mapsto Mk_2$. Both cases produce an overall factor of M^{n-2} . One then expands the remaining integrands in M , interchanges summation and integration, and adds up the two contributions. Since the integral in Eq. (3) is invariant under the interchange of k_1 and k_2 , we only need to perform one of the above manipulations and then multiply the result with a factor of 2. As a result we obtain

$$G(p^2, m^2, M^2, n) = -\frac{g^2}{(2\pi)^{2n}} \sum_{i,j=0}^{\infty} \sum_{a=0}^j \sum_{b=0}^a (-1)^j 2^{j-b} \binom{j}{a} \binom{a}{b} M^{2i+j+b} I_{ij,ab}(p^2, m^2, n), \quad (9)$$

where

$$\binom{r}{s} = \frac{r!}{s!(r-s)!}$$

is a binomial coefficient and the integrals $I_{ij,ab}(p^2, m^2, n)$ are defined as

$$I_{ij,ab}(p^2, m^2, n) = \iint \frac{d^n k_1 d^n k_2 (p \cdot k_1)^{j-a} (k_1 \cdot k_2)^{a-b} (k_1^2)^b}{(k_1^2 - 1 + i0^+)(k_2^2 + i0^+)^{1+i} [(p + k_2)^2 - m^2 + i0^+]^{1+j}}. \quad (10)$$

³ In the IR renormalization there is an infinite number of subtraction terms which contribute in higher orders; in the following, however, only terms up to the order of the given calculation will be explicitly shown.

For $j + b$ odd, the loop integral of Eq. (10) vanishes, because in that case the integrand is an odd function of k_1 . From this one concludes that the nonvanishing terms in Eq. (9) are proportional to nonnegative integer powers of M^2 . Using a dimensional counting analysis [36], it can be shown that $I_{ij,ab}(p^2, m^2, n)$ is of the form

$$I_{ij,ab}(p^2, m^2, n) = \sum_{l=0}^{\infty} (p^2 - m^2)^l g_{ij,ab,l}^{(1)}(m^2, n) + (p^2 - m^2)^{n+a-b-2i-j-3} \sum_{l=0}^{\infty} (p^2 - m^2)^l g_{ij,ab,l}^{(2)}(m^2, n). \quad (11)$$

Combining this result with Eqs. (4) and (9), we see that the term nonanalytic in $p^2 - m^2$ is of order Q^{2n-5+a} ($a \geq 0$) and therefore does not violate the power counting. On the other hand, the first part of Eq. (11) contains terms that are analytic in $p^2 - m^2$ which, when combined with Eqs. (4) and (9), give rise to contributions which are nonanalytic in M :

$$M^{n-2}G^{(1)} = -g^2 M^{n-2} \frac{m^{n-4}\Gamma(2-n/2)}{(4\pi)^{n/2}(n-3)} \left[1 - \frac{1}{2m^2}(p^2 - m^2) \right] \frac{\Gamma(1-n/2)}{(4\pi)^{n/2}} + \dots \quad (12)$$

Note that the first term in Eq. (12) violates the power counting and, since it is nonanalytic in M , cannot be directly absorbed by a counterterm. In solving this apparent puzzle, one has to keep in mind that, in order to consistently renormalize the diagram of Fig. 1 (a), all relevant contributions from counterterms in \mathcal{L}_1 of Eq. (1) must be taken into account. In the present case, these counterterms originate in the renormalization of the one-loop diagrams of Fig. 2 (a) and (b) with the corresponding expressions

$$\mathcal{M}_{2a} = g^2 \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - M^2 + i0^+)[(k+p+q)^2 - m^2 + i0^+]} \quad (13)$$

and

$$\mathcal{M}_{2b} = g^2 \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - M^2 + i0^+)[(k+p-q')^2 - m^2 + i0^+]}, \quad (14)$$

respectively. Both diagrams are assigned the order Q^{n-3} . Evaluating Eqs. (13) and (14) we find that \mathcal{M}_{2a} and \mathcal{M}_{2b} contain contributions that violate the power counting. To renormalize these diagrams we follow Refs. [10, 14] and, up to order Q , find the subtraction terms as

$$\Delta\mathcal{M}_{2a}^{\text{EOMS}} + \Delta\mathcal{M}_{2b}^{\text{EOMS}} = 2ig^2\lambda(m, n) \quad (15)$$

and

$$\Delta\mathcal{M}_{2a}^{\text{IR}} + \Delta\mathcal{M}_{2b}^{\text{IR}} = ig^2\lambda(m, n) \left\{ 2 - \frac{1}{2m^2}[(p+q)^2 + (p-q')^2 - 2m^2] \right\}, \quad (16)$$

where

$$\lambda(m, n) = \frac{m^{n-4}\Gamma(2-n/2)}{(4\pi)^{n/2}(n-3)}. \quad (17)$$

The counterterms are then given by

$$\mathcal{L}_{CT}^{\text{EOMS}} = -\frac{1}{2}g^2\lambda(m, n)\pi^2\Psi^2 \quad (18)$$

for the EOMS scheme and by

$$\mathcal{L}_{CT}^{\text{IR}} = -\frac{1}{2}g^2\lambda(m, n) \left[\pi^2\Psi^2 + \frac{1}{2m^2} \left(\pi^2\Psi\partial_\mu\partial^\mu\Psi + 2\pi\partial_\mu\pi\Psi\partial^\mu\Psi + \pi\partial_\mu\partial^\mu\pi\Psi^2 + m^2\pi^2\Psi^2 \right) \right] \quad (19)$$

in the IR renormalization. In a two-loop calculation these counterterms give a contribution to the self-energy as shown in Fig. 1 (b). The corresponding expressions read

$$-i\Sigma_{CT}^{\text{EOMS}} = -\frac{1}{2}i \int \frac{d^n k}{(2\pi)^n} \left(\Delta\mathcal{M}_{2a}^{\text{EOMS}} + \Delta\mathcal{M}_{2b}^{\text{EOMS}} \right) \frac{1}{k^2 - M^2 + i0^+} = -ig^2\lambda(m, n) I_\pi, \quad (20)$$

and

$$\begin{aligned} -i\Sigma_{CT}^{\text{IR}} &= -\frac{1}{2}i \int \frac{d^n k}{(2\pi)^n} \left(\Delta\mathcal{M}_{2a}^{\text{IR}} + \Delta\mathcal{M}_{2b}^{\text{IR}} \right) \frac{1}{k^2 - M^2 + i0^+} \\ &= -ig^2\lambda(m, n) \left[1 - \frac{p^2 - m^2}{2m^2} - \frac{M^2}{2m^2} \right] I_\pi, \end{aligned} \quad (21)$$

where

$$I_\pi = i \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 - M^2 + i0^+} \sim Q^2. \quad (22)$$

In fact, the last term in Eq. (21) is of order Q^4 ,

$$M^2 I_\pi \sim Q^4, \quad (23)$$

and can therefore be neglected in a calculation up to and including order Q^3 . Noting that

$$I_\pi = M^{n-2} \frac{\Gamma(1 - n/2)}{(4\pi)^{n/2}}, \quad (24)$$

we see that in the IR renormalization the self-energy contribution of Eq. (21), which stems from the renormalization of one-loop diagrams, exactly cancels the contributions of Eq. (12) which are explicitly shown, including the part which violates the power counting. In the EOMS scheme no terms that satisfy the power counting are subtracted.

Let us finally consider the last term of Eq. (4). The function $H(p^2, m^2, M^2, n)$ is obtained from Eq. (3) by simultaneously rescaling $k_1 \mapsto Mk_1$ and $k_2 \mapsto Mk_2$, extracting a factor of M^{2n-4} , expanding the remaining integrand in M , and interchanging integration and summation, yielding

$$\begin{aligned} H(p^2, m^2, M^2, n) &= -\frac{g^2}{2(2\pi)^{2n}} \frac{1}{p^2 - m^2} \sum_{i=0}^{\infty} (-1)^i \left(\frac{M}{p^2 - m^2} \right)^i \\ &\quad \times \sum_{j=0}^i \binom{i}{j} \iint d^n k_1 d^n k_2 \frac{[2p \cdot (k_1 + k_2)]^{i-j} [M(k_1 + k_2)^2]^j}{(k_1^2 - 1 + i0^+)(k_2^2 - 1 + i0^+)}. \end{aligned} \quad (25)$$

Noting that $M/(p^2 - m^2) \sim Q^0$, it is easy to see that Eq. (25) in combination with the factor M^{2n-4} of Eq. (4) satisfies the power counting. Furthermore, only nonnegative integer powers of M^2 survive in H .

Combining the results above, all terms violating the power counting are canceled in the sum of the diagrams in Fig. 1 in both the IR and the EOMS renormalization. Finally, we have also studied the case where first $p^2 - m^2 \rightarrow 0$ and then $M \rightarrow 0$, and have explicitly verified that the renormalization procedure remains exactly the same, i.e., the counterterms are the same and the renormalized diagram satisfies the power counting.

IV. CONCLUSION

In conclusion, we have demonstrated that the application of both the reformulated IR renormalization and the EOMS scheme to two-loop diagrams leads to a consistent power counting. In this context, the subtraction of one-loop sub-diagrams plays an important role in the renormalization of two-loop diagrams. The procedure can be applied iteratively to multiloop diagrams. Calculations using the Lagrangian of baryon chiral perturbation theory are more involved, but the general features of the renormalization program do not change.

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V. APPENDIX

In this appendix we provide an illustration of the dimensional counting analysis of Ref. [36] in terms of a specific example. To that end let us consider the integral of Eq. (13) for $q = 0$:

$$I_{\pi\Psi}(0, p) = \frac{i}{(2\pi)^n} \int \frac{d^n k}{(k^2 - M^2 + i0^+)[(k + p)^2 - m^2 + i0^+]}. \quad (26)$$

One would like to know how the integral behaves for small values of M and/or $p^2 - m^2$ as a function of n . If we consider, for fixed $p^2 \neq m^2$, the limit $M \rightarrow 0$, the integral $I_{\pi\Psi}(0, p)$ can be represented as

$$I_{\pi\Psi}(0, p) = \sum_i M^{\beta_i} F_i(p^2, m^2, M^2, n), \quad (27)$$

where the functions F_i are analytic in M^2 and are obtained as follows. First, one rewrites the variable of the loop integration as $k = M^{a_i} \tilde{k}$, where a_i is an arbitrary nonnegative real number. Next, one isolates the overall factor of M^{β_i} so that the remaining integrand can be expanded in positive powers of M and interchanges the integration and summation. The resulting series represents the expansion of $F_i(p^2, m^2, M^2, n)$ in powers of M^2 . The summation over i includes all values of i for which F_i is nontrivial. To be specific, we obtain for $I_{\pi\Psi}(0, p)$:

$$I_{\pi\Psi}(0, p) = \frac{i}{(2\pi)^n} \int \frac{M^{na_i} d^n \tilde{k}}{[\tilde{k}^2 M^{2a_i} - M^2 + i0^+][\tilde{k}^2 M^{2a_i} + 2p \cdot \tilde{k} M^{a_i} + p^2 - m^2 + i0^+]}. \quad (28)$$

From Eq. (28) we see that the second propagator does not contribute to the overall factor M^{β_i} for any a_i and has to be expanded in (positive) powers of $(\tilde{k}^2 M^{2a_i} + 2p \cdot \tilde{k} M^{a_i})$ unless $a_i = 0$. For $0 < a_i < 1$, we rewrite the first propagator as

$$\frac{1}{M^{2a_i}} \frac{1}{(\tilde{k}^2 - M^{2-2a_i} + i0^+)} \quad (29)$$

and expand the second factor in Eq. (29) in positive powers of M^{2-2a_i} . On the other hand, if $1 < a_i$ we write the first propagator as

$$\frac{1}{M^2} \frac{1}{(\tilde{k}^2 M^{2a_i-2} - 1 + i0^+)} \quad (30)$$

and expand the second factor in Eq. (30) in positive powers of M^{2a_i-2} . In both cases one obtains integrals of the type $\int d^n \tilde{k} \tilde{k}^\alpha$ as the coefficients of the expansion. However, such integrals vanish in dimensional regularization. Therefore the only nontrivial terms in the sum of Eq. (27) correspond to either $a_i = 0$ or $a_i = 1$. Thus we obtain

$$I_{\pi\Psi}(0, p) = I_{\pi\Psi}^{(0)}(0, p) + I_{\pi\Psi}^{(1)}(0, p), \quad (31)$$

where

$$I_{\pi\Psi}^{(0)}(0, p) = \frac{i}{(2\pi)^n} \sum_{i=0}^{\infty} (M^2)^i \int \frac{d^n k}{[k^2 + i0^+]^{1+i} [k^2 + 2p \cdot k + p^2 - m^2 + i0^+]}, \quad (32)$$

and

$$I_{\pi\Psi}^{(1)}(0, p) = \frac{i}{(2\pi)^n} \sum_{i=0}^{\infty} \frac{M^{n-2}(-1)^i}{p^2 - m^2} \left(\frac{M}{p^2 - m^2} \right)^i \int \frac{d^n \tilde{k} (\tilde{k}^2 M + 2p \cdot \tilde{k})^i}{[\tilde{k}^2 - 1 + i0^+]}. \quad (33)$$

On the other hand, the integral $I_{\pi\Psi}(0, p)$ can be calculated directly without applying the dimensional counting technique. Doing so and comparing with the results given above one sees that the dimensional counting method leads to the correct expressions.

While the loop integrals of Eq. (33) have a simple analytic structure in $p^2 - m^2$, the technique sketched above can be directly applied to the loop integrals of Eq. (32) when $p^2 - m^2 \rightarrow 0$, now using the change of variable $k = (p^2 - m^2)^{c_j} \tilde{k}$ with arbitrary nonnegative real numbers c_j .

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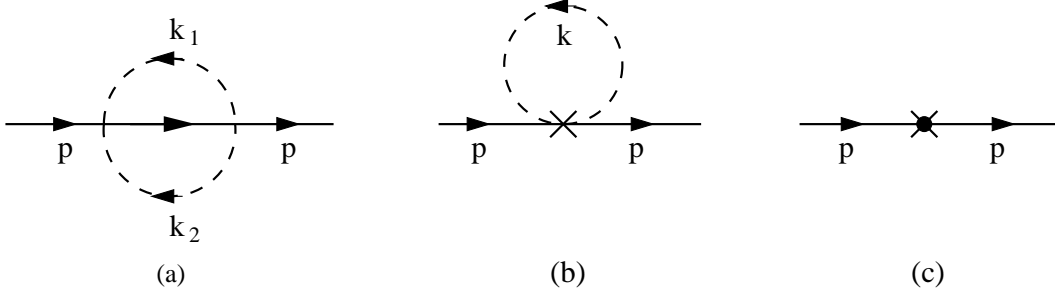


FIG. 1: Diagrams contributing in two-loop order to the self-energy of Ψ . A simple cross corresponds to one-loop order counterterms and a cross with a dot corresponds to two-loop order counterterms.

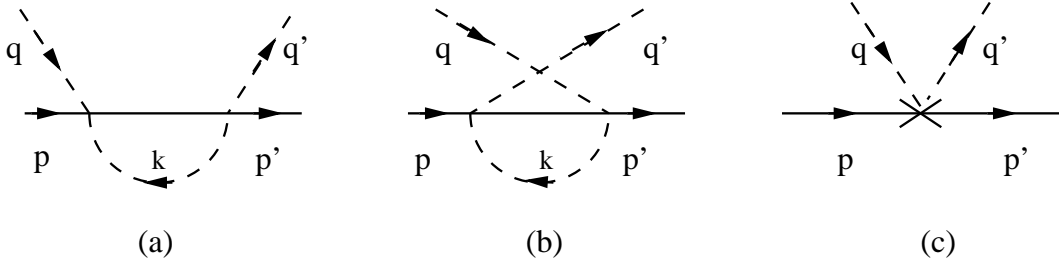


FIG. 2: Diagrams contributing to $\pi\Psi$ scattering.